Factor Based Dirichlet Portfolios

Purushottam Parthasarathy, IEOR, IITB

Feb 27, 2023

Outline

- Motivation for the problem
- Problem Setting

Outline

- Motivation for the problem
- Problem Setting
- Prior work

Outline

- Motivation for the problem
- Problem Setting
- Prior work
- Our Work
- Results
- Summary

Motivation and History

- Two major schools of thought in portfolio engineering -

Motivation and History

- Two major schools of thought in portfolio engineering -
- First is the Markowitz style of one period optimization with covariance and return information

Motivation and History

- Two major schools of thought in portfolio engineering -
- First is the Markowitz style of one period optimization with covariance and return information
- Second is the online portfolio growth theory stream initiated by Kelly and Cover

Motivation and History

- Two major schools of thought in portfolio engineering -
- First is the Markowitz style of one period optimization with covariance and return information
- Second is the online portfolio growth theory stream initiated by Kelly and Cover
- Online portfolio growth theory to obtain long term bounds on investment algorithms in a multi-period setting - an explicit optimization step may or may not be involved

Motivation and History

- Two major schools of thought in portfolio engineering -
- First is the Markowitz style of one period optimization with covariance and return information
- Second is the online portfolio growth theory stream initiated by Kelly and Cover
- Online portfolio growth theory to obtain long term bounds on investment algorithms in a multi-period setting - an explicit optimization step may or may not be involved
- Our goal is to use techniques from operations research and industrial engineering to attempt to improve the state of the art in the latter

Online Portfolio Growth Theory - Kelly Betting

- Kelly(1956) gave the first expression for the optimal proportion of capital to be invested if the investor wants to maximize long term growth rate of her portfolio

Online Portfolio Growth Theory - Kelly Betting

- Kelly(1956) gave the first expression for the optimal proportion of capital to be invested if the investor wants to maximize long term growth rate of her portfolio
  - Kelly is asking how much capital to risk in each round of investment when investor has a private wire to the results of a horse race

Online Portfolio Growth Theory - Kelly Betting

- Kelly(1956) gave the first expression for the optimal proportion of capital to be invested if the investor wants to maximize long term growth rate of her portfolio
  - Kelly is asking how much capital to risk in each round of investment when investor has a private wire to the results of a horse race
  - Problem setup is an double or nothing game

Online Portfolio Growth Theory - Kelly Betting

- Kelly(1956) gave the first expression for the optimal proportion of capital to be invested if the investor wants to maximize long term growth rate of her portfolio
  - Kelly is asking how much capital to risk in each round of investment when investor has a private wire to the results of a horse race
  - Problem setup is an double or nothing game
  - The result is simple, invest $l = 2q - 1$ where l is proportion of capital and q is probability that private signal is correct

Online Portfolio Growth Theory - Kelly Betting

- Kelly(1956) gave the first expression for the optimal proportion of capital to be invested if the investor wants to maximize long term growth rate of her portfolio
  - Kelly is asking how much capital to risk in each round of investment when investor has a private wire to the results of a horse race
  - Problem setup is an double or nothing game
  - The result is simple, invest $l = 2q - 1$ where l is proportion of capital and q is probability that private signal is correct
  - Example, if q is 0.5, you have no edge and you don't invest!

Online Portfolio Growth Theory - Kelly Betting

- Kelly(1956) gave the first expression for the optimal proportion of capital to be invested if the investor wants to maximize long term growth rate of her portfolio
  - Kelly is asking how much capital to risk in each round of investment when investor has a private wire to the results of a horse race
  - Problem setup is an double or nothing game
  - The result is simple, invest $l = 2q - 1$ where l is proportion of capital and q is probability that private signal is correct
  - Example, if q is 0.5, you have no edge and you don't invest!
  - Note that this is the first attempt to obtain results to maximize long term growth rate which is $\frac{1}{N} log \frac{V_N}{V_0}$, where V_N is value of the portfolio after N rounds

Online Portfolio Growth Theory - Constant Rebalanced Portfolios

- Cover(1984) proposed an algorithm to calculate the best re-balanced portfolio without the benefit of hindsight

Online Portfolio Growth Theory - Constant Rebalanced Portfolios

- Cover(1984) proposed an algorithm to calculate the best re-balanced portfolio without the benefit of hindsight
  - Given a vector of multiplicative returns **X** where X_i is the proportional price improvement (or decrement) of asset i and **b** is the portfolio vector defined on the unit simplex such that $b \geq 0$, the expected log-return of the investor after one period would be: $E(\log b^t X)$

Online Portfolio Growth Theory - Constant Rebalanced Portfolios

- Cover(1984) proposed an algorithm to calculate the best re-balanced portfolio without the benefit of hindsight
  - Given a vector of multiplicative returns **X** where X_i is the proportional price improvement (or decrement) of asset i and **b** is the portfolio vector defined on the unit simplex such that $b \geq 0$, the expected log-return of the investor after one period would be: $E(\log b^t X)$
  - The goal is to find the best b^* along multiple investment periods, and the algorithm proposed by Cover is

Online Portfolio Growth Theory - Constant Rebalanced Portfolios

- Cover(1984) proposed an algorithm to calculate the best re-balanced portfolio without the benefit of hindsight
  - Given a vector of multiplicative returns **X** where X_i is the proportional price improvement (or decrement) of asset i and **b** is the portfolio vector defined on the unit simplex such that $b \geq 0$, the expected log-return of the investor after one period would be: $E(\log b^t X)$
  - The goal is to find the best b^* along multiple investment periods, and the algorithm proposed by Cover is

$$b_i^{'} = b_i \frac{X_i}{b^t X}, b^0 > 0 \tag{1}$$

Online Portfolio Growth Theory - Constant Rebalanced Portfolios

- Cover(1984) proposed an algorithm to calculate the best re-balanced portfolio without the benefit of hindsight
  - Given a vector of multiplicative returns **X** where X_i is the proportional price improvement (or decrement) of asset i and **b** is the portfolio vector defined on the unit simplex such that $b \geq 0$, the expected log-return of the investor after one period would be: $E(\log b^t X)$
  - The goal is to find the best b^* along multiple investment periods, and the algorithm proposed by Cover is

$$b_i^{'} = b_i \frac{X_i}{b^t X}, b^0 > 0 \tag{1}$$

  - Where b^0 is chosen strictly inside the boundary of the simplex. We can see the update step is a multiplicative update to move the portfolio vector along the gradient of the log return function shown above.

Online Portfolio Growth Theory - Constant Rebalanced Portfolios

- Cover(1984) proposed an algorithm to calculate the best re-balanced portfolio without the benefit of hindsight
  - Given a vector of multiplicative returns **X** where X_i is the proportional price improvement (or decrement) of asset i and **b** is the portfolio vector defined on the unit simplex such that $b \geq 0$, the expected log-return of the investor after one period would be: $E(\log b^t X)$
  - The goal is to find the best b^* along multiple investment periods, and the algorithm proposed by Cover is

$$b_i^{'} = b_i \frac{X_i}{b^t X}, b^0 > 0 \tag{1}$$

  - Where b^0 is chosen strictly inside the boundary of the simplex. We can see the update step is a multiplicative update to move the portfolio vector along the gradient of the log return function shown above.
  - Cover shows that in the limit $b^{'} \rightarrow b^*$ which must exist as it is the result of the maximization of a concave function over the simplex

Online Portfolio Growth Theory - Universal Portfolios I

- Cover(1996) proposed an improved algorithm or portfolio strategy to calculate the ‘universal’ portfolio that asymptotically approaches the best re-balanced portfolio, without the benefit of hindsight

Online Portfolio Growth Theory - Universal Portfolios I

- Cover(1996) proposed an improved algorithm or portfolio strategy to calculate the ‘universal’ portfolio that asymptotically approaches the best re-balanced portfolio, without the benefit of hindsight
- As defined before $x = (x_1, x_2, ..., x_m)^t \geq 0$ denotes stock market vector for one investment period, where x_i is the price relative for one stock

Online Portfolio Growth Theory - Universal Portfolios I

- Cover(1996) proposed an improved algorithm or portfolio strategy to calculate the ‘universal’ portfolio that asymptotically approaches the best re-balanced portfolio, without the benefit of hindsight
- As defined before $x = (x_1, x_2, ..., x_m)^t \geq 0$ denotes stock market vector for one investment period, where x_i is the price relative for one stock
- A portfolio $b = (b_1, b_2, b_3, ...b_m)^t \geq 0$ s.t $\Sigma b_i = 1$ is the proportion current wealth invested in each of the m stocks

Online Portfolio Growth Theory - Universal Portfolios I

- Cover(1996) proposed an improved algorithm or portfolio strategy to calculate the 'universal' portfolio that asymptotically approaches the best re-balanced portfolio, without the benefit of hindsight
- As defined before $x = (x_1, x_2, ..., x_m)^t \geq 0$ denotes stock market vector for one investment period, where x_i is the price relative for one stock
- A portfolio $b = (b_1, b_2, b_3, ...b_m)^t \geq 0$ s.t $\Sigma b_i = 1$ is the proportion current wealth invested in each of the m stocks
- Define total wealth that is re-invested repeatedly as $S_n(b) = \prod_{i=1}^{n} b^t x_i$

Online Portfolio Growth Theory - Universal Portfolios I

- Cover(1996) proposed an improved algorithm or portfolio strategy to calculate the 'universal' portfolio that asymptotically approaches the best re-balanced portfolio, without the benefit of hindsight
- As defined before $x = (x_1, x_2, ..., x_m)^t \geq 0$ denotes stock market vector for one investment period, where x_i is the price relative for one stock
- A portfolio $b = (b_1, b_2, b_3, ...b_m)^t \geq 0$ s.t $\Sigma b_i = 1$ is the proportion current wealth invested in each of the m stocks
- Define total wealth that is re-invested repeatedly as $S_n(b) = \prod_{i=1}^{n} b^t x_i$
- The best possible re-balanced portfolio is that Cover sets up as a benchmark is $S_n^* = \max_b S_n(b)$

Online Portfolio Growth Theory - Universal Portfolios II

- Cover's idea is setting up an infinite number of portfolio managers in the unit simplex, distributed **uniformly**, each given allocation b

Online Portfolio Growth Theory - Universal Portfolios II

- Cover's idea is setting up an infinite number of portfolio managers in the unit simplex, distributed **uniformly**, each given allocation b
- The performance or total wealth of any manager at the end of k periods is is $S_k = \prod_{i=1}^{k} b^t x_i$

Online Portfolio Growth Theory - Universal Portfolios II

- Cover's idea is setting up an infinite number of portfolio managers in the unit simplex, distributed **uniformly**, each given allocation b
- The performance or total wealth of any manager at the end of k periods is is $S_k = \prod_{i=1}^{k} b^t x_i$
- Consider a weighted allocation that rewards the managers who do well and proportionally penalizes the ones that do poorly, and integrate over the simplex, a weighted portfolio: $\frac{\int bS(b)db}{\int S(b)db}$

Online Portfolio Growth Theory - Universal Portfolios II

- Cover's idea is setting up an infinite number of portfolio managers in the unit simplex, distributed **uniformly**, each given allocation b
- The performance or total wealth of any manager at the end of k periods is is $S_k = \prod_{i=1}^{k} b^t x_i$
- Consider a weighted allocation that rewards the managers who do well and proportionally penalizes the ones that do poorly, and integrate over the simplex, a weighted portfolio: $\frac{\int bS(b)db}{\int S(b)db}$
- Key result, the portfolio that approximates the best constant re-balance portfolio with hindsight is the same weighted portfolio:

Online Portfolio Growth Theory - Universal Portfolios II

- Cover's idea is setting up an infinite number of portfolio managers in the unit simplex, distributed **uniformly**, each given allocation b
- The performance or total wealth of any manager at the end of k periods is is $S_k = \prod_{i=1}^{k} b^t x_i$
- Consider a weighted allocation that rewards the managers who do well and proportionally penalizes the ones that do poorly, and integrate over the simplex, a weighted portfolio: $\frac{\int bS(b)db}{\int S(b)db}$
- Key result, the portfolio that approximates the best constant re-balance portfolio with hindsight is the same weighted portfolio:

$$b_{k+1} = \frac{\int_{\Delta} bS_k(b)db}{\int_{\Delta} S_k(b)db} \tag{2}$$

where

$$S_k(b) = \prod_{i=1}^{k} b^t x_i \tag{3}$$

Online Portfolio Growth Theory - Universal Portfolios With Side Information

- Cover and Ordentlich(1996) give another proposal for dealing with 'side information'

Online Portfolio Growth Theory - Universal Portfolios With Side Information

- Cover and Ordentlich(1996) give another proposal for dealing with ‘side information’
- Side information is a discrete variable that is revealed before the portfolio is constructed every period, like a signal or a news item

Online Portfolio Growth Theory - Universal Portfolios With Side Information

- Cover and Ordentlich(1996) give another proposal for dealing with 'side information'
- Side information is a discrete variable that is revealed before the portfolio is constructed every period, like a signal or a news item
- This extends to the concept of a state-constant rebalanced portfolio, in the dimension k(m-1) where k is number of possible states for the side information and m is the number of assets

Online Portfolio Growth Theory - Universal Portfolios With Side Information

- Cover and Ordentlich(1996) give another proposal for dealing with 'side information'
- Side information is a discrete variable that is revealed before the portfolio is constructed every period, like a signal or a news item
- This extends to the concept of a state-constant rebalanced portfolio, in the dimension k(m-1) where k is number of possible states for the side information and m is the number of assets
- They show that we can construct parallel histories, one for each discrete variable and deploy the universal portfolio from that history

Online Portfolio Growth Theory - Universal Portfolios With Side Information

- Cover and Ordentlich(1996) give another proposal for dealing with 'side information'
- Side information is a discrete variable that is revealed before the portfolio is constructed every period, like a signal or a news item
- This extends to the concept of a state-constant rebalanced portfolio, in the dimension k(m-1) where k is number of possible states for the side information and m is the number of assets
- They show that we can construct parallel histories, one for each discrete variable and deploy the universal portfolio from that history
- For a uniform and Dirichlet(1/2, 1/2, 1/2...1/2) distribution on the simplex of portfolios, the authors show that the above portfolio is 'universal' - i.e. the growth achieved by this portfolio is asymptotically the same as the best state-constant rebalanced portfolio.

Online Portfolio Growth Theory - Solving approximations to the Cover Formulation

- In Helmbold, Schapire, Singer, Warmuth(1996) authors use a utility function that serves to trade off a performance measure and a distance measure. $F(W^{t+1}) = \eta log(W^{t+1}x^t) - d(W^{t+1}, W_t)$

Online Portfolio Growth Theory - Solving approximations to the Cover Formulation

- In Helmbold, Schapire, Singer, Warmuth(1996) authors use a utility function that serves to trade off a performance measure and a distance measure. $F(W^{t+1}) = \eta log(W^{t+1}x^{t}) - d(W^{t+1}, W_{t})$
- The performance is based on x_t, the current relative price vector and the distance measure d is chosen to be the relative entropy between the new weight vector and the current one.

Online Portfolio Growth Theory - Solving approximations to the Cover Formulation

- In Helmbold, Schapire, Singer, Warmuth(1996) authors use a utility function that serves to trade off a performance measure and a distance measure. $F(W^{t+1}) = \eta log(W^{t+1}x^t) - d(W^{t+1}, W_t)$
- The performance is based on x_t, the current relative price vector and the distance measure d is chosen to be the relative entropy between the new weight vector and the current one.
- An approximation of the optimization objective function along with a full invested constraint leads to a recursive update algorithm that uses a exponential update to the weight vector.

Online Portfolio Growth Theory - Solving approximations to the Cover Formulation

- In Helmbold, Schapire, Singer, Warmuth(1996) authors use a utility function that serves to trade off a performance measure and a distance measure. $F(W^{t+1}) = \eta log(W^{t+1}x^t) - d(W^{t+1}, W_t)$
- The performance is based on x_t, the current relative price vector and the distance measure d is chosen to be the relative entropy between the new weight vector and the current one.
- An approximation of the optimization objective function along with a full invested constraint leads to a recursive update algorithm that uses a exponential update to the weight vector.
- In various empirical tests, the algorithm achieves better out of sample performance than Cover's portfolio and some other benchmarks

Literature Post 1996 (I)

- Hence Helmbold et. al.(1996) show that exponential multiplicative updates yield almost the same performance as the universal portfolio and yet have less memory requirements.

Literature Post 1996 (I)

- Hence Helmbold et. al.(1996) show that exponential multiplicative updates yield almost the same performance as the universal portfolio and yet have less memory requirements.
- Some approaches use a contrarian approach

Literature Post 1996 (I)

- Hence Helmbold et. al.(1996) show that exponential multiplicative updates yield almost the same performance as the universal portfolio and yet have less memory requirements.
- Some approaches use a contrarian approach
- In one such work the authors in Borodin, El-Yaniv, Gogan(2003) show that a forecast step that uses mean-reversion can be used to get better portfolios. The authors calculate a transfer(asset i to asset j) which depends upon the following factors:

Literature Post 1996 (I)

- Hence Helmbold et. al.(1996) show that exponential multiplicative updates yield almost the same performance as the universal portfolio and yet have less memory requirements.
- Some approaches use a contrarian approach
- In one such work the authors in Borodin, El-Yaniv, Gogan(2003) show that a forecast step that uses mean-reversion can be used to get better portfolios. The authors calculate a transfer(asset i to asset j) which depends upon the following factors:
  - Non zero only if asset i has performed better than asset j in window W
  - The strength of the transfer between i and j is proportion to the positive correlation between asset i and j in a lead lag setup (effectively asset i leads asset j in window W)
  - Strength is also proportional to the negative auto-correlation of asset i to itself

Literature Post 1996 (I)

- Hence Helmbold et. al.(1996) show that exponential multiplicative updates yield almost the same performance as the universal portfolio and yet have less memory requirements.
- Some approaches use a contrarian approach
- In one such work the authors in Borodin, El-Yaniv, Gogan(2003) show that a forecast step that uses mean-reversion can be used to get better portfolios. The authors calculate a transfer(asset i to asset j) which depends upon the following factors:
  - Non zero only if asset i has performed better than asset j in window W
  - The strength of the transfer between i and j is proportion to the positive correlation between asset i and j in a lead lag setup (effectively asset i leads asset j in window W)
  - Strength is also proportional to the negative auto-correlation of asset i to itself
- The above work is an example where a heuristic is demonstrated to better than prior technique, including Cover's universal portfolio

Literature Post 1996 (II)

- There is also a set of work that has been done that explores meta-algorithms(MAs)

Literature Post 1996 (II)

- There is also a set of work that has been done that explores meta-algorithms(MAs)
- Das and Banerjee use daily SP500 data for the past 21 years to show that MAs outperform existing portfolio selection algorithms by several orders of magnitude.

Literature Post 1996 (II)

- There is also a set of work that has been done that explores meta-algorithms(MAs)
- Das and Banerjee use daily SP500 data for the past 21 years to show that MAs outperform existing portfolio selection algorithms by several orders of magnitude.
- Li et al. have written a comprehensive review paper that describes 4 major techniques in this area:

Literature Post 1996 (II)

- There is also a set of work that has been done that explores meta-algorithms(MAs)
- Das and Banerjee use daily SP500 data for the past 21 years to show that MAs outperform existing portfolio selection algorithms by several orders of magnitude.
- Li et al. have written a comprehensive review paper that describes 4 major techniques in this area:
    1. Follow the winner type approaches (universal portfolios(1994), exponential gradient(Helmbold et al. 1996))
    2. Follow the loser approaches (antiCor(Borodin et al. 2003), robust median reversion(Hyung et al. 2013), regularized mean reversion(Li et al. 2015))
    3. Pattern Matching (Find returns that match current returns and forecast - Gyorfi et al. 2007)
    4. Meta Algorithms (Das and Bannerjee(2011) - which combine other algorithms and perform an online gradient or online newton method update)

Our Work: Preliminaries (I)

- Cover's work involves Dirichlet uniform sampling which depends on α, the concentration parameter

Our Work: Preliminaries (I)

- Cover's work involves Dirichlet uniform sampling which depends on α, the concentration parameter
- Consider a uniform α of $(1/2,\ 1/2,\ 1/2)$, sample three variables from it 200 times:

Our Work: Preliminaries (I)

- Cover's work involves Dirichlet uniform sampling which depends on α, the concentration parameter
- Consider a uniform α of $(1/2, 1/2, 1/2)$, sample three variables from it 200 times:

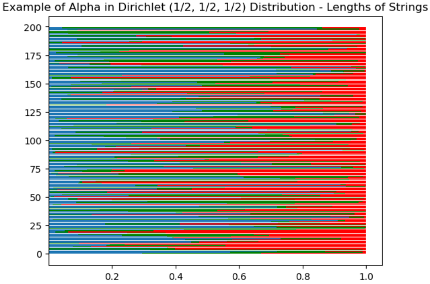

Figure 1: Dirichlet$(1/2, 1/2, 1/2)$

Our Work: Preliminaries (I)

- Cover's work involves Dirichlet uniform sampling which depends on α, the concentration parameter
- Consider a uniform α of $(1/2, 1/2, 1/2)$, sample three variables from it 200 times:

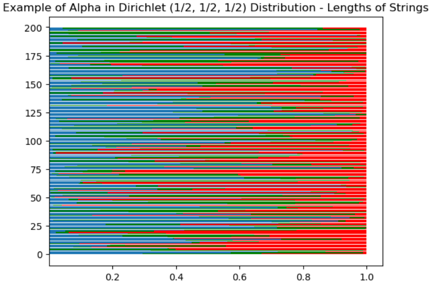

Figure 1: Dirichlet$(1/2,\ 1/2,\ 1/2)$

- We can see a uniform weight given to the three variables

Our Work: Preliminaries (II)

- Now consider Dirichlet(10, 100 ,200)

Our Work: Preliminaries (II)

- Now consider Dirichlet(10, 100 ,200)

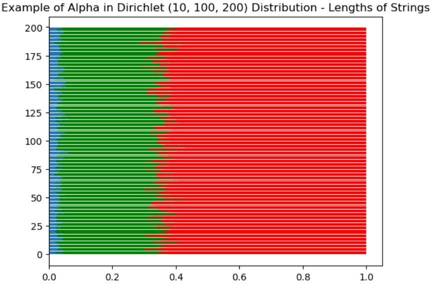

Figure 2: Dirichlet(10, 100, 200)

Our Work: Preliminaries (II)

- Now consider Dirichlet(10, 100 ,200)

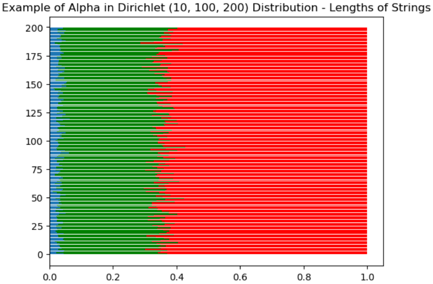

Figure 2: Dirichlet(10, 100, 200)

- So the α parameter can be abstracted as a 'belief' of the portfolio manager

Our Work: Dirichlet Factor Portfolios

- Lot of literature on factors that drive equity returns, most famous being the Fama French three-factor model(1992) who modify the CAPM model to add two more factors to explain cross section of equity returns: $R = r + \beta(R_M - r_f) + \beta_{SMB}SMB + \beta_{HML}HML + \epsilon$

Our Work: Dirichlet Factor Portfolios

- Lot of literature on factors that drive equity returns, most famous being the Fama French three-factor model(1992) who modify the CAPM model to add two more factors to explain cross section of equity returns: $R = r + \beta(R_M - r_f) + \beta_{SMB}SMB + \beta_{HML}HML + \epsilon$
- Jegadish and Titman(1993) show that momentum is also a factor in explaining the cross section of equity returns

Our Work: Dirichlet Factor Portfolios

- Lot of literature on factors that drive equity returns, most famous being the Fama French three-factor model(1992) who modify the CAPM model to add two more factors to explain cross section of equity returns: $R = r + \beta(R_M - r_f) + \beta_{SMB}SMB + \beta_{HML}HML + \epsilon$
- Jegadish and Titman(1993) show that momentum is also a factor in explaining the cross section of equity returns
- Post this work 5-factor models have emerged, and funds dedicate entire teams to factor research, thereby making this line of research very applied from an industry stand-point

Our Work: Dirichlet Factor Portfolios

- Lot of literature on factors that drive equity returns, most famous being the Fama French three-factor model(1992) who modify the CAPM model to add two more factors to explain cross section of equity returns: $R = r + \beta(R_M - r_f) + \beta_{SMB}SMB + \beta_{HML}HML + \epsilon$
- Jegadish and Titman(1993) show that momentum is also a factor in explaining the cross section of equity returns
- Post this work 5-factor models have emerged, and funds dedicate entire teams to factor research, thereby making this line of research very applied from an industry stand-point
- Our idea is to combine Cover's work with prior beliefs that portfolio managers might have

Our Work: Dirichlet Factor Portfolios

- Lot of literature on factors that drive equity returns, most famous being the Fama French three-factor model(1992) who modify the CAPM model to add two more factors to explain cross section of equity returns: $R = r + \beta(R_M - r_f) + \beta_{SMB}SMB + \beta_{HML}HML + \epsilon$
- Jegadish and Titman(1993) show that momentum is also a factor in explaining the cross section of equity returns
- Post this work 5-factor models have emerged, and funds dedicate entire teams to factor research, thereby making this line of research very applied from an industry stand-point
- Our idea is to combine Cover's work with prior beliefs that portfolio managers might have
- Our algorithm hence biases the dirichlet alpha parameter based on any generic factor and calculates the performance weighted universal portfolio from these biased portfolio managers

Our Work: Dirichlet Factor Portfolios

- Lot of literature on factors that drive equity returns, most famous being the Fama French three-factor model(1992) who modify the CAPM model to add two more factors to explain cross section of equity returns: $R = r + \beta(R_M - r_f) + \beta_{SMB}SMB + \beta_{HML}HML + \epsilon$
- Jegadish and Titman(1993) show that momentum is also a factor in explaining the cross section of equity returns
- Post this work 5-factor models have emerged, and funds dedicate entire teams to factor research, thereby making this line of research very applied from an industry stand-point
- Our idea is to combine Cover's work with prior beliefs that portfolio managers might have
- Our algorithm hence biases the dirichlet alpha parameter based on any generic factor and calculates the performance weighted universal portfolio from these biased portfolio managers

Our Work: Dirichlet Factor Portfolios

Algorithm 1 Calculating portfolio weights for size based dirichlet portfolio

1: **while** $t \neq T$ **do**

Our Work: Dirichlet Factor Portfolios

Algorithm 2 Calculating portfolio weights for size based dirichlet portfolio

1: **while** $t \neq T$ **do**

2: $\alpha_{jt} \leftarrow 1/p_{jt} \forall j \in 1..M$

Our Work: Dirichlet Factor Portfolios

Algorithm 3 Calculating portfolio weights for size based dirichlet portfolio

while $t \neq T$ **do**
  $\alpha_{jt} \leftarrow 1/p_{jt} \forall j \in 1..M$
  $\alpha_{jt} \leftarrow \frac{\alpha_{jt}}{min(\alpha_{jt})} \forall j \in 1..M$
  $\alpha_{jt} \leftarrow \max(1, \log \alpha_{jt}) \forall j \in 1..M$

Our Work: Dirichlet Factor Portfolios

Algorithm 4 Calculating portfolio weights for size based dirichlet portfolio

1: **while** $t \neq T$ **do**

2: $\alpha_{jt} \leftarrow 1/p_{jt} \forall j \in 1..M$

3: $\alpha_{jt} \leftarrow \frac{\alpha_{jt}}{min(\alpha_{jt})} \forall j \in 1..M$

4: $\alpha_{jt} \leftarrow \max(1, \log \alpha_{jt}) \forall j \in 1..M$

Require: Simulate N-Managers dirichlet portfolios from α_t

Our Work: Dirichlet Factor Portfolios

Algorithm 5 Calculating portfolio weights for size based dirichlet portfolio

1: **while** $t \neq T$ **do**

2: $\alpha_{jt} \leftarrow 1/p_{jt} \forall j \in 1..M$

3: $\alpha_{jt} \leftarrow \frac{\alpha_{jt}}{min(\alpha_{jt})} \forall j \in 1..M$

4: $\alpha_{jt} \leftarrow \max(1, \log \alpha_{jt}) \forall j \in 1..M$

Require: Simulate N-Managers dirichlet portfolios from α_t

Require: Calculate Universal Portfolios from factor biased managers

5: $w_{jt} \leftarrow \frac{\int (Sbdb)}{\int (Sdb)}$

Our Work: Dirichlet Factor Portfolios

Algorithm 6 Calculating portfolio weights for size based dirichlet portfolio

1: **while** $t \neq T$ **do**
2: $\alpha_{jt} \leftarrow 1/p_{jt} \forall j \in 1..M$
3: $\alpha_{jt} \leftarrow \frac{\alpha_{jt}}{min(\alpha_{jt})} \forall j \in 1..M$
4: $\alpha_{jt} \leftarrow \max(1, \log \alpha_{jt}) \forall j \in 1..M$
Require: Simulate N-Managers dirichlet portfolios from α_t
Require: Calculate Universal Portfolios from factor biased managers
5: $w_{jt} \leftarrow \frac{\int(Sbdb)}{\int(Sdb)}$
Require: Evaluate performance of calculated weight vector
6: $p_{t+1} = w_t X_{t+1}$
7: $t \leftarrow t + 1$
8: **end while**

Our Work: Dirichlet Factor Portfolios

Algorithm 7 Calculating portfolio weights for size based dirichlet portfolio

while $t \neq T$ **do**
    $\alpha_{jt} \leftarrow 1/p_{jt} \forall j \in 1..M$
    $\alpha_{jt} \leftarrow \frac{\alpha_{jt}}{min(\alpha_{jt})} \forall j \in 1..M$
    $\alpha_{jt} \leftarrow \max(1, \log \alpha_{jt}) \forall j \in 1..M$
Require: Simulate N-Managers dirichlet portfolios from α_t
Require: Calculate Universal Portfolios from factor biased managers
    $w_{jt} \leftarrow \frac{\int(Sbdb)}{\int(Sdb)}$
Require: Evaluate performance of calculated weight vector
    $p_{t+1} = w_t X_{t+1}$
    $t \leftarrow t + 1$
end while

This can be varied appropriately for any factor, technical or fundamental

Cover's Portfolio vs Factor Dirichlet Portfolios - NASDAQ

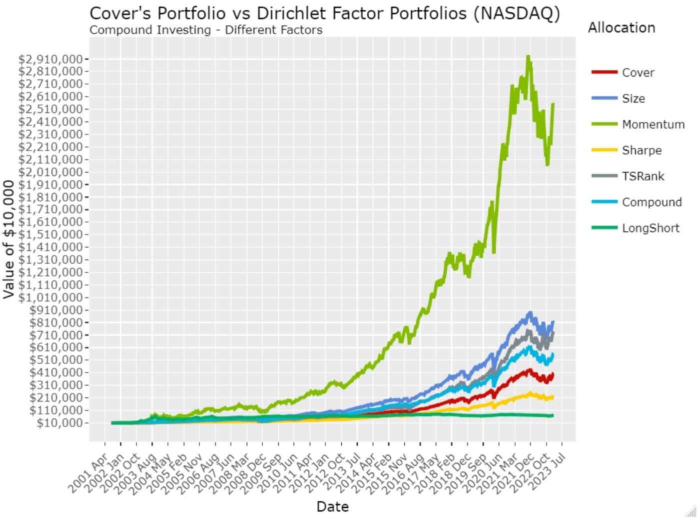

Figure 3: Different Factors Using NASDAQ Constituents

Cover's Portfolio vs Factor Dirichlet Portfolios - FTSE

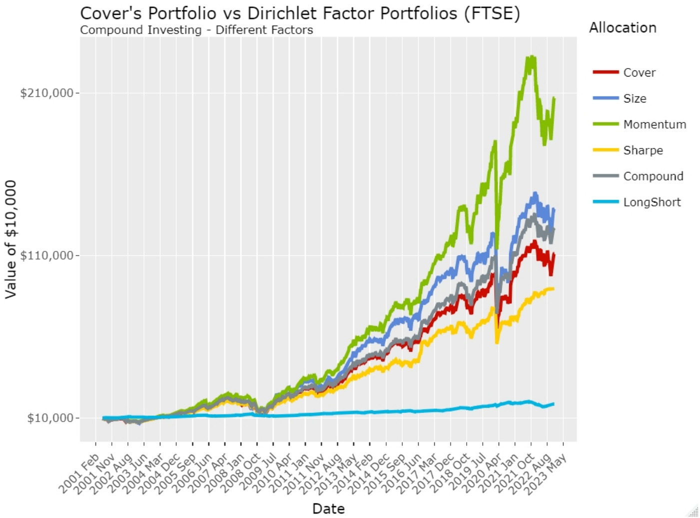

Figure 4: Different Factors Using FTSE Constituents

Summary I

- Online portfolio optimization is a rich area of research where there is around 70 years of literature

Summary I

- Online portfolio optimization is a rich area of research where there is around 70 years of literature
- We are attempting to combine economic insight and the work of information theorists to offer a practical heuristic that is easy to compute and implement

Summary I

- Online portfolio optimization is a rich area of research where there is around 70 years of literature
- We are attempting to combine economic insight and the work of information theorists to offer a practical heuristic that is easy to compute and implement
- This technique is flexible in that it can be used to combine different views into compound portfolios

Summary II

- Our reward for last year!

Summary II

- Our reward for last year!

Figure 5: Bhardwaj, Avinash, Manjesh K. Hanawal, and Purushottam Parthasarathy. "Almost Exact Risk Budgeting with Return Forecasts for Portfolio Allocation." Operations Research Letters (2023).